Homework 1 problems:

2.5a, 2.17, 3.1 a, d, e, 3.9,

Determine whether e-x2 is convex, pseudoconvex or quasiconvex.

Homework 2 solutions.

2.5a. S={x12+x32≤x2}. Observe that for a fixed x2, there is a circle going perpendicular to the x2 plane. Thus, the interior is {x12+x32<x2}, The boundary is {x12+x32=x2}. The closure is just the set S={x12+x32≤x2}.

2.17 Let C1 and C2 be convex cones. Show that conv(C1 ) is a convex cone.

Clearly, conv(C1 ) is convex. Now let dє conv(C1 ). We need to show that λd є conv(C1 ) for all λ≥0.

Since dє conv(C1 ), there exists a d1 є C1 and d2 є C2 and an α є {0,1} such that α d1 +(1-α) d2 =d. Since d1 є C1 and d2 є C2, then λd1 є C1 and λd2 є C2 by definition of a cone for any λ≥0. Thus, α λd1 +(1-α)λ d2 є conv(C1 ) for any λ≥0. So λ(α d1 +(1-α) d2 ) =λd є conv(C1 ) and the result follows.

3.1a

f(x1,x2) = 2 x12 -4 x1x2-8x1+ 3x2

The gradient of f is | 4x1-4 x2-8 |

| -4x1+3 |

So the hessian is |4 -4 |

|-4 0 |

This is neither convex nor concave.

3c.

f(x1,x2) = - x12 - 3x22+4 x1x2+10x1 -10x2

The gradient of f is | -2x1+4 x2+10 |

| -6x2+4x1-10 |

So the hessian is |-2 4 |

|4 -6 |

All diagonals are negative, so it can be at best a concave function.

(-2)(-6)-4\*4=-4. So this function is neither. (The last one must be positive pg 120).

3d.

f(x1,x2,x3) = 2x1 x2  +2x12+ x22+ 2x32-5 x1x3

The gradient of f is | 2x2+4 x1-5x3 |

| 2x1+2 x2 |

| 4x3-5x1 |

So the hessian is |4 2 -5 |

|2 2 0 |

|-5 0 4 |

No noticeable inconsistencies, so perform one ERO.

|4 2 -5 |

|0 1 2.5 |

|0 2.5 -9/4 |

This is neither convex nor concave.

3.9 Prove that if f1, f2, …,fk are convex functions that max { f1, f2, …,fk} is convex.

Assume f1 and f2 are convex. Thus, any x1 and x2 and any αє(0,1) we have

f1(α x1 + (1-α) x2) ≤ α f1(x1)+ (1-α) f1(x2) and

f2(α x1 + (1-α) x2) ≤ α f2(x1)+ (1-α) f2(x2). Clearly, we get that

max { f1(α x1 + (1-α) x2), f2(α x1 + (1-α) x2)} ≤ max { α f1(x1)+ (1-α) f1(x2), α f2(x1)+ (1-α) f2(x2)}. Thus, if f’=max {f1,f2}, then f’ is convex.

By applying this result to f’, f’’=max {f’, f3}, it is easy to see that f’’ is convex. Thus max { f1, f2, …,fk} is convex and the proof is complete.

Easton 1.

Determine whether e-x2 is convex, pseudoconvex or quasiconvex.

Take x=0 and x=1 and α=.5. Then we get .5e0 +.5 e-1 = .6839. However, e-.52 =.778. Thus, the function is larger than the line segment connecting 0 and 1and the function is not convex.

Given two points x1 and x2, wlog (without loss of generality) assume | x1 | ≥ |x2|, thus the max of e-.x12 and of e-.x22 occurs at x2 with a value of e-.x22. Let’s examine f(α x1 +(1-α) x2) = e-(αx1 .+(1-α)x2)2for any αє (0,1). Clearly, e-(αx1 .+(1-α)x2)2 = e -α2x12e -2α(1-α)x1 x2e –(1-α)2x22≤ e -α2x22e -2α(1-α)x2 x2e –(1-α)2x22= e -x22. Thus, the function is quasi convex.

f’= -2x e -x2. Examine x1 =-1 and x2 = 2. -2x1 e –x12 (x2-x1) = 2e-1(2- -1) = 4>0. However, f(-1) = e-1 > f(2) = e-4 and so f is not pseudoconvex.